Noise Part 1: Time and Frequency Analysis
Noise Part 2:Transitor-level analysis

Analog ICs

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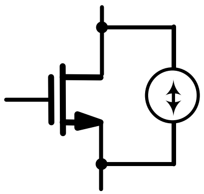
Chap2-Outline1 : Noise in Devices

- MOSFET Channel Thermal Noise
- MOSFET Flicker Noise
- Circuit-Noise Analysis and Modeling
  - Example: Noise and SNR in CS Amplifier
- Input-Referred Noise
  - Effect of source impedance
- Examples: Noise analyses in classical single-stage amplifiers (CS, CG, and CS).

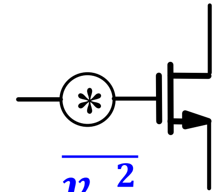
Channel Thermal Noise



$$\overline{i_n^2} = 4kT\gamma g_m \ [A^2/\ Hz]$$

(in saturation)

<u>Channel current thermal noise</u>



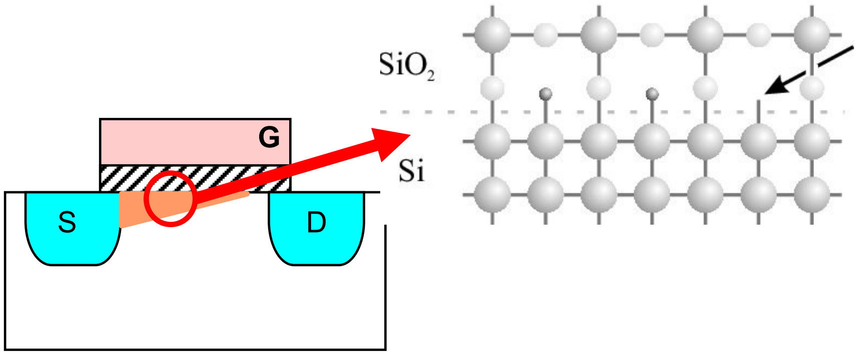
$$\overline{v_n^2} = 4kT\gamma \frac{1}{g_m} \ [V^2/\ Hz]$$

<u>Input Voltage representation</u>

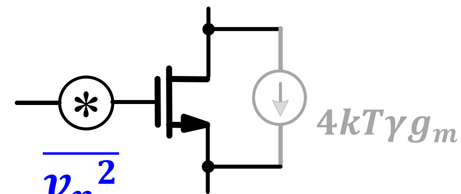
- With $\gamma \approx 2/3$ for long channel MOSFETs (→1 for modern technologies)
- nMOS has a larger g_m ($\mu_e > \mu_h$) and so larger channel current thermal noise

 Should we minimize or maximize g_m?

Flicker Noise

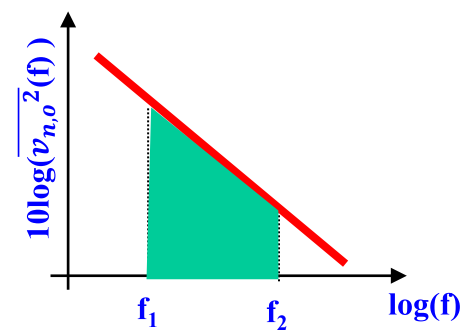


Dangling Bonds
→ Trapping and de-trapping mobile carriers
→ Flicker noise



$$\overline{v_n{}^2} = \frac{K}{C_{ox}LW}\cdot\frac{1}{f} \quad [V^2/Hz]$$

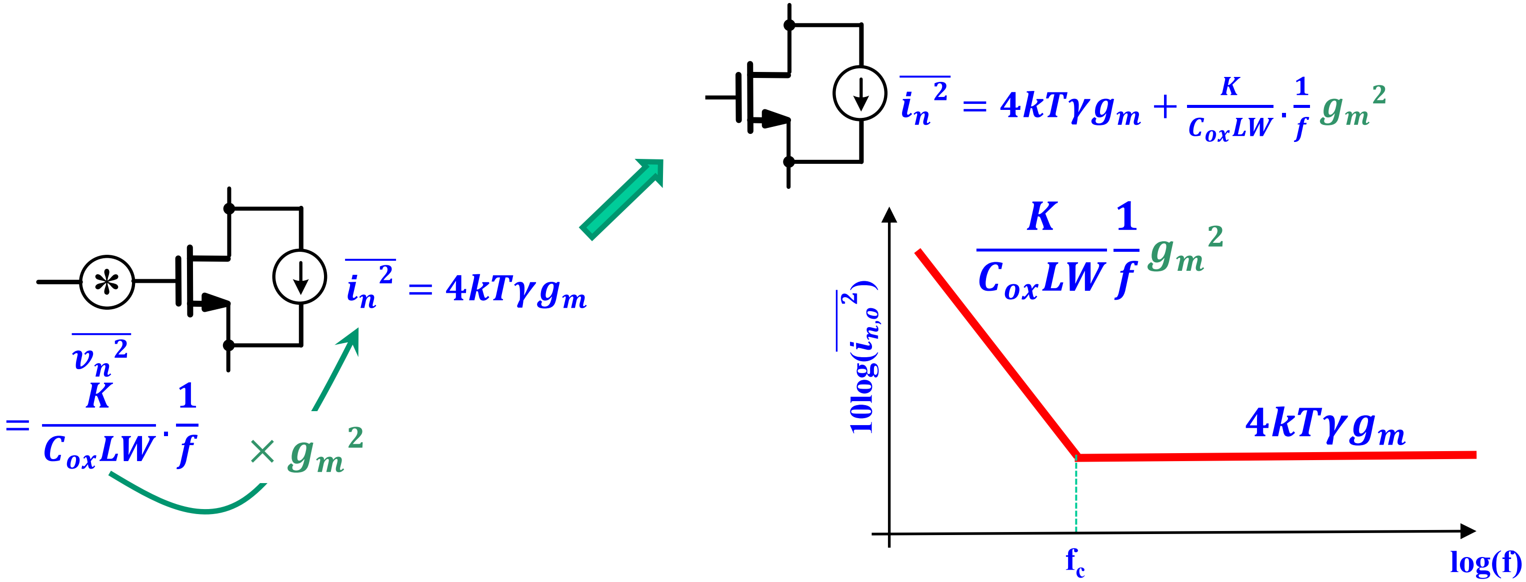
- K is a constant empirically determined
- Big transistors have less flicker noise
- K is generally lower for PMOS



- Ex: The total noise power from f_1 to f_2 [V^2]:

$$\overline{v_n{}^2} = \int_{f_1}^{f_2} \frac{K}{C_{ox}LW}\cdot\frac{1}{f}\,df \quad = \frac{K}{C_{ox}LW}\cdot Ln\left(\frac{f_2}{f_1}\right) \quad [V^2]$$

Total Noise in MOSFETs

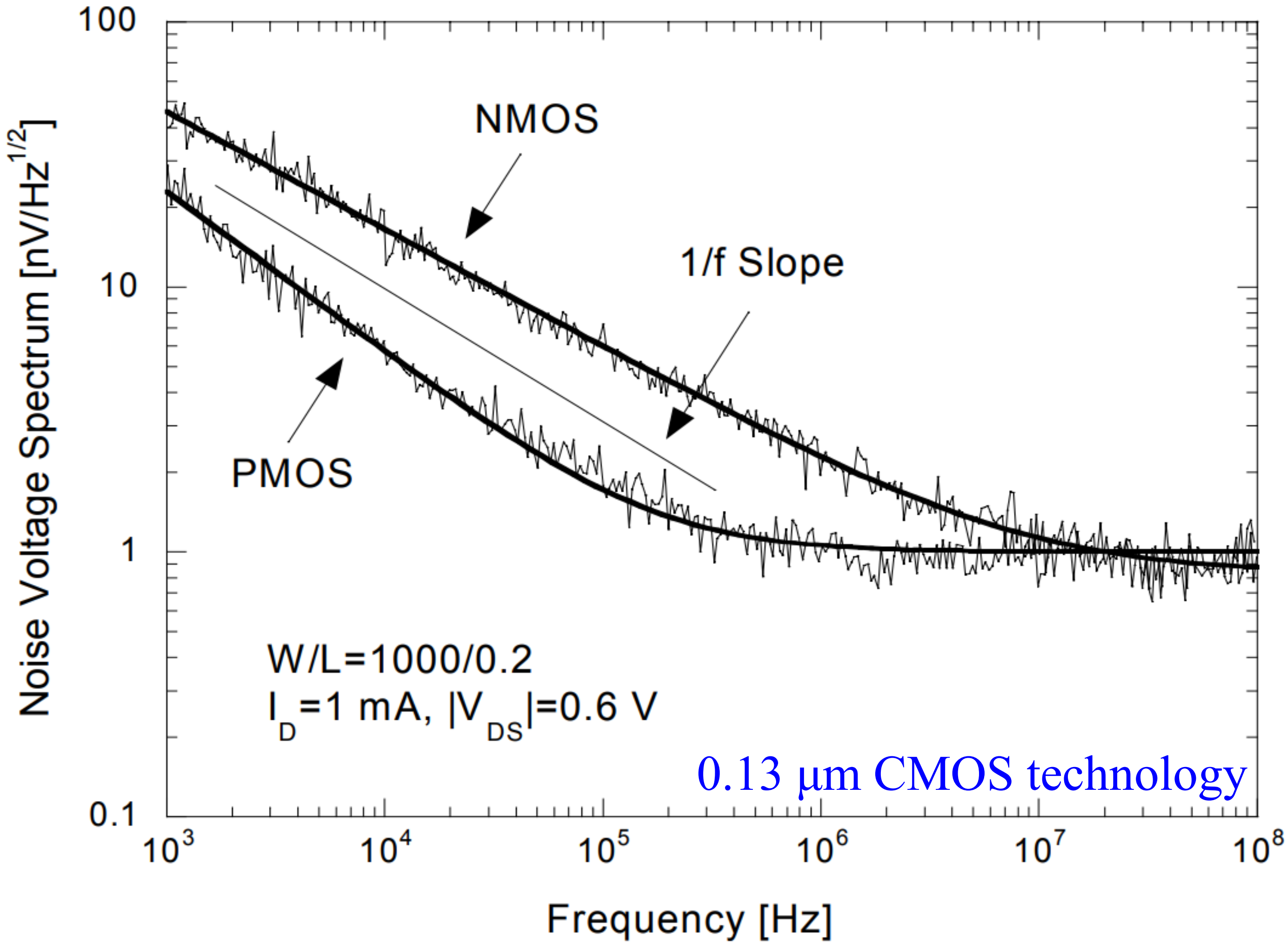


f_c: Flicker noise "corner frequency"

$$f_c = \frac{K}{c_{ox}WL} g_m \frac{3}{8kT}$$ (usually between 500 kHz, 1 MHz)

- Flicker noise is detrimental for low-frequency circuits
- Or if it is upconverted the signal Band.

nMOS vs pMOS



NMOS
1/f Slope
PMOS
W/L=1000/0.2
I_D=1 mA, |V_DS|=0.6 V
0.13 μm CMOS technology
Noise Voltage Spectrum [nV/Hz^1/2]
100
10
1
0.1
10^3
10^4
10^5
10^6
10^7
10^8
Frequency [Hz]

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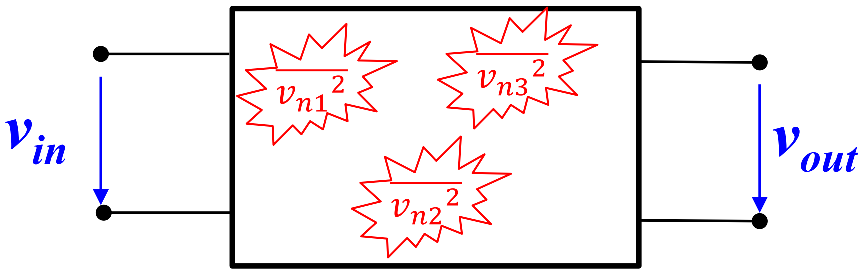
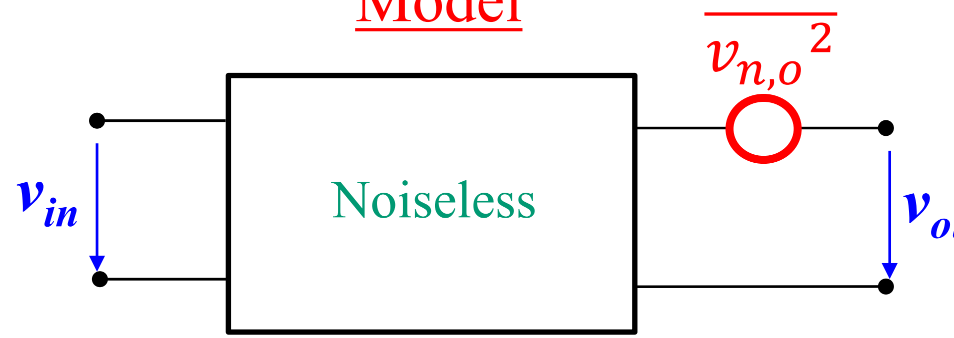
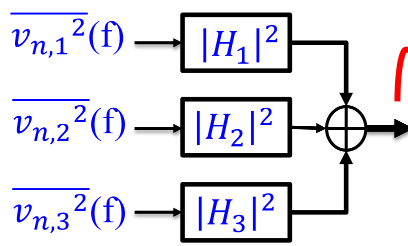
Circuit-Noise Analysis and Modeling Procedure



Model



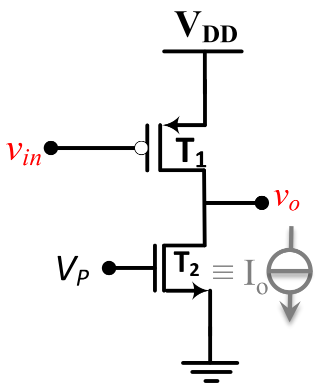
- Identify noise sources.
- Determine ($H_i(s)$) from $\overline{v_{ni}^2}$ to the output.
- Add all of the contributions to the output



$$\overline{v_{n,o}^2}(f) = \sum_{j=1}^{3} \overline{v_{n,j}^2}(f)\,\left|H_j(f)\right|^2$$

Example 1: Noise in CS Amplifier

- Determine the output thermal noise voltage of a common source amplifier and deduce the impact of g_{m1} and g_{m2} on noise performance.



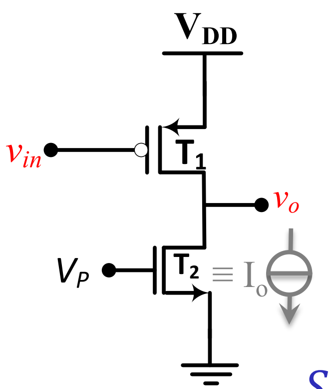
$$\overline{i_{n1,2}{}^2} = 4kT\gamma g_{m1,2} \;[A^2/\,Hz]$$

$$\overline{v_{n,o}{}^2} = 4kT\gamma(g_{m1} + g_{m1})(r_{o1}//r_{o2})^2 \;[V^2/\,Hz]$$

Output Noise ↘ if g_{m1} ↘ and g_{m2} ↘

Example 2: SNR of a CS Amplifier

- Revaluate the noise performance using the signal-to-noise ratio (SNR).



$$|A_v| = g_{m1}\, r_{o1} // r_{o2}$$

$$\overline{v_{n,o}^{\,2}} = 4kT\gamma(g_{m1} + g_{m2})(r_{o1} // r_{o2})^2 \ [V^2/ Hz]$$

$$SNR = \frac{\overline{v_o^{\,2}}}{\overline{v_{n,o}^{\,2}}} = \frac{|A_v|^2\overline{v_i^{\,2}}}{\overline{v_{n,o}^{\,2}}} = \frac{g_{m1}^{\,2}}{4kT\gamma(g_{m1}+g_{m2})\Delta f}\overline{v_i^{\,2}} = \frac{1}{\Delta f}\frac{g_{m1}}{4kT\gamma\left(1+\frac{g_{m2}}{g_{m1}}\right)}\overline{v_i^{\,2}}$$

$SNR \nearrow$ if $g_{m1} \nearrow$ and $g_{m2} \searrow$

Conclusion: improve noise performances → maximize g_m of active transistor and minimize g_m of load (current source)

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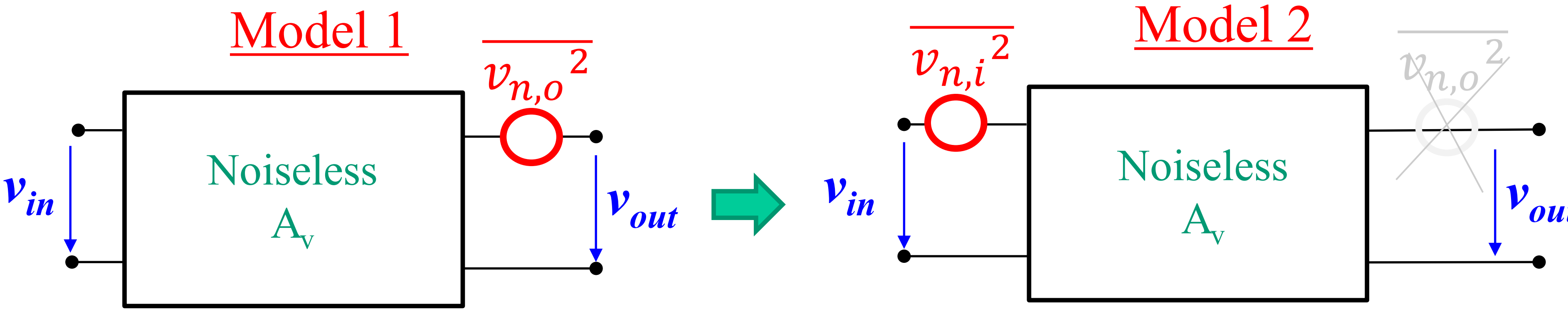
Input-Referred Noise



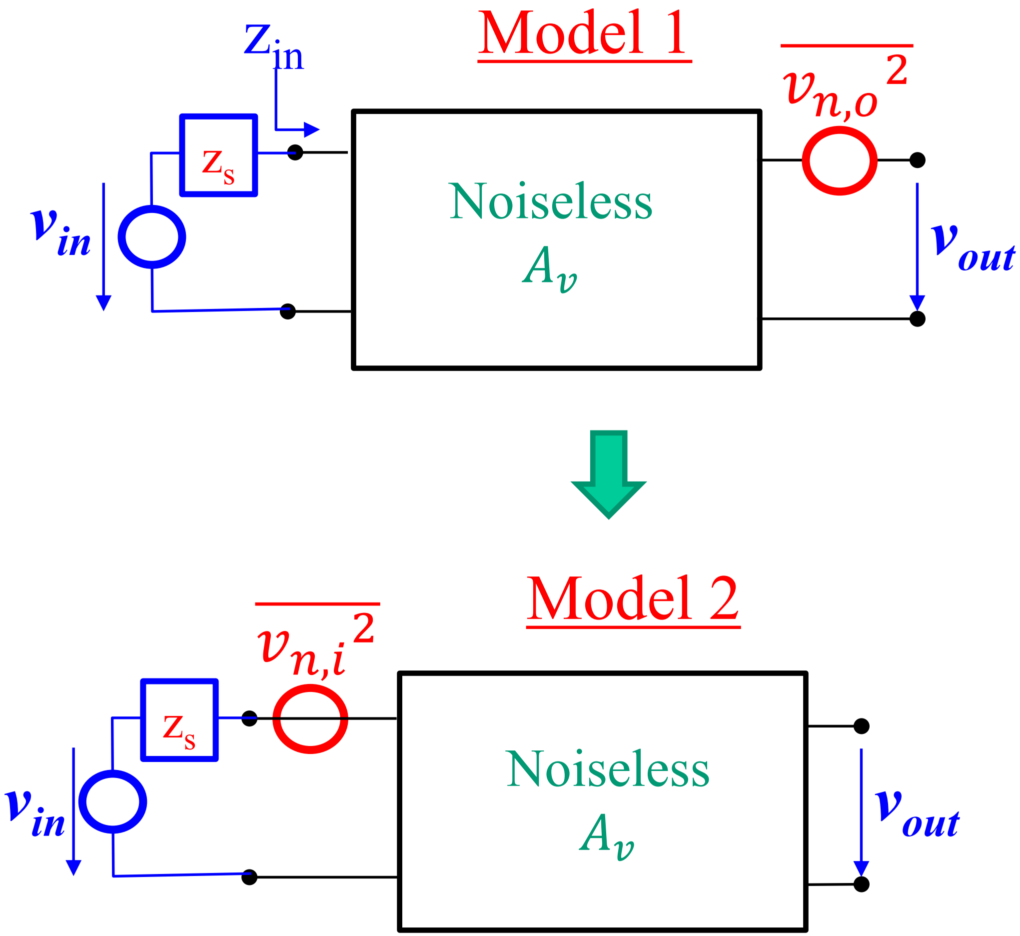
A_v is the voltage gain

$$\overline{v_{n,i}^{2}} = \frac{\overline{v_{n,o}^{2}}}{A_v^{2}}$$

- Why is $\overline{v_{n,i}^{2}}$ important?:
  - Optimizing $\overline{v_{n,o}^{2}}$ can degrade the gain (A_v) and so the desired signal

  → Better optimize $\overline{v_{n,i}^{2}}$ (just like SNR)

Effect of source Impedance on Input-Referred Noise



$$\overline{v_{n,i}^2} = \frac{\overline{v_{n,o}^2}}{\left(A_v \frac{z_{in}}{z_{in}+z_s}\right)^2}$$

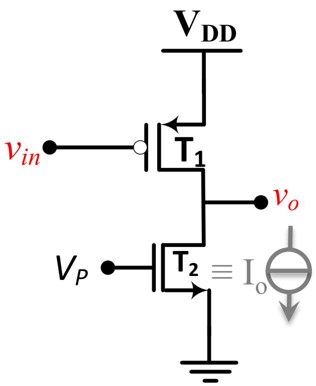
Note:

z_s have no effect if $z_{in} \rightarrow \infty$

(e.g. CS and CD at low-frequency)

Example 3: Input-referred Noise in CS Amplifier

- Determine the input-referred voltage noise of CS amp at low frequency (flicker and thermal).



$$|A_v| = g_{m1}\, r_{o1} // r_{o2}$$

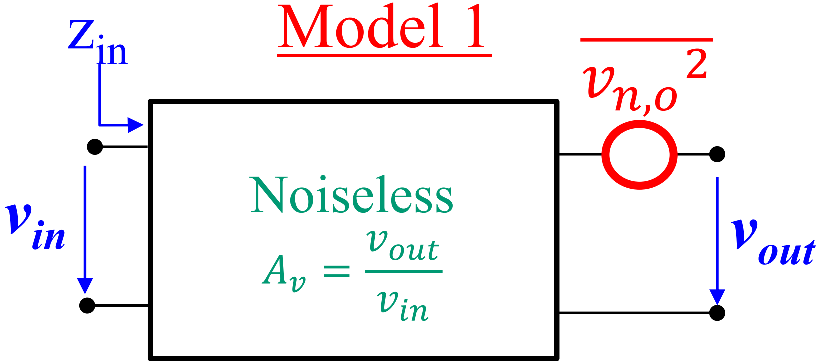
$$\overline{v_{n,o}^2} = 4kT\gamma(g_{m1} + g_{m2})(r_{o1} // r_{o2})^2 + |A_v|^2 \frac{K}{C_{ox}(LW)_1} \cdot \frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2 |A_v|^2 \frac{K}{C_{ox}(LW)_2} \cdot \frac{1}{f}$$

$$\overline{v_{n,i}^2} = \frac{\overline{v_{n,o}^2}}{|A_v|^2} = 4kT\gamma \frac{1}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right) + \frac{K}{C_{ox}(LW)_1} \cdot \frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \frac{K}{C_{ox}(LW)_2} \cdot \frac{1}{f} \quad [V^2/Hz]$$

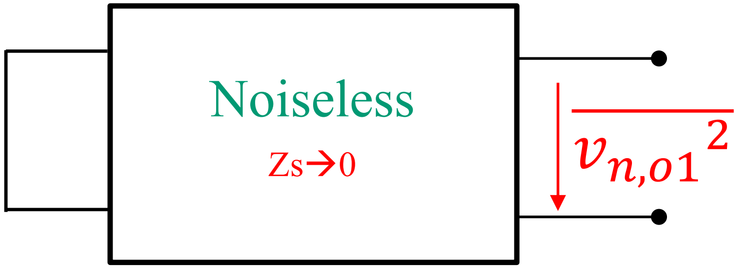
Conclusion: improve noise performances → maximize g_m of active transistor and minimize g_m of load (current source)

Complete Input-Referred Noise Model : Valid $\forall z_s$

Model 1

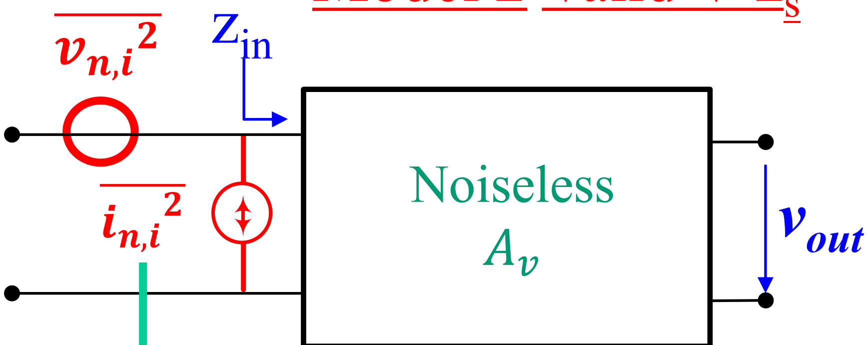


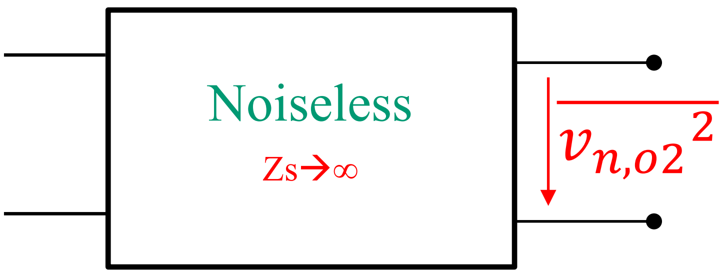
A_v is the voltage gain



$$\overline{v_{n,i}^2} = \frac{\overline{v_{n,o1}^2}}{(A_v)^2}$$

Model 2 Valid $\forall$ z_s





$$\overline{i_{n,i}^2} = \frac{\overline{v_{n,o2}^2}}{(z_{in}A_v)^2}$$

Correlated

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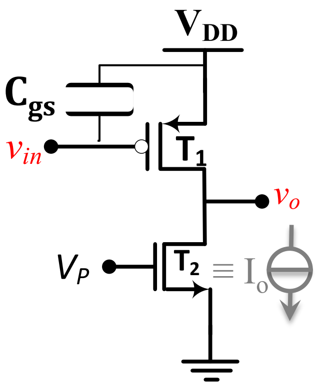
Example 3: Noise in CS Amplifier

- Determine the input-referred current noise of CS amp at high frequency (consider Cgs capacitor).

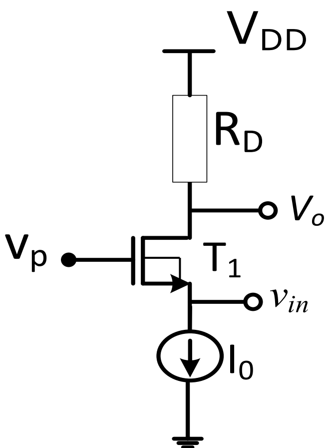


$$Z_{in} = \frac{1}{j2\pi f C_{gs}}$$

$$\overline{i_{n,i}^{\,2}} = \frac{\overline{v_{no,2}^{\,2}}}{|Z_{in}A_v|^2} = \left(2\pi f C_{gs}\right)^2 4kT\gamma \frac{1}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right) + \left(2\pi f C_{gs}\right)^2 \left(\frac{g_{m2}}{g_{m1}}\right)^2 \frac{K}{C_{ox}(LW)_2} \cdot \frac{1}{f} \ [\mathrm{A^2/\ Hz}]$$

Example 4: Noise in CG Amplifier

- Determine the output and input-referred thermal noise voltage and current of a CG amp (neglect the noise of I_0).



$$|A_v| = g_{m1}\, r_{o1} // R_D \approx g_{m1} R_D$$

$$z_{in} = \frac{1}{g_{m1}}\left(1 + \frac{R_D}{r_{o1}}\right) // \, r_{o2} \approx \frac{1}{g_{m1}}$$

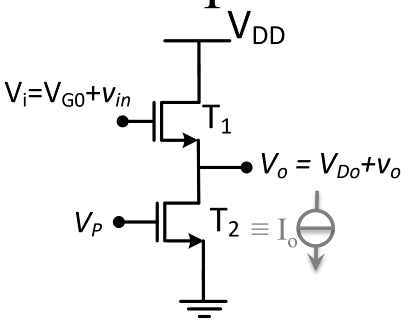
$$\overline{{v_{n,0}}^2} = 4kT\gamma g_{m1} {R_D}^2 + 4kTR_D$$

$$\overline{{v_{n,i}}^2} = \frac{\overline{{v_{n,o1}}^2}}{|A_v|^2} = 4kT\gamma \frac{1}{g_{m1}} + 4kT \frac{1}{R_D (g_{m1})^2} \quad (\text{short-circuit the input})$$

$$\overline{{i_{n,i}}^2} = \frac{\overline{{v_{n,o2}}^2}}{|z_{in} A_v|^2} = 4kT \frac{1}{R_D} \quad (\text{open the input})$$

Example 5: Noise in CD Amplifier (Source Follower)

- Determine the output and input-referred thermal noise of a Source Follower at low frequency (flicker and thermal) and comment on its noise performances.



$$|A_v| = 1$$

$$\overline{v_{n,o}^2} = 4kT\gamma(g_{m1}+g_{m2})\left(\frac{1}{g_{m1}}\right)^2 + \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

$$\overline{v_{n,i}^2} = \frac{\overline{v_{n,o}^2}}{|A_v|^2} = 4kT\gamma\frac{1}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right) + \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2\frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f} \quad [V^2/Hz]$$

Note: $\overline{v_{n,i}^2}$ of CD is similar to CS but with no gain for the desired signal